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# Abstracts of Papers to Appear in Future Issues

COMPUTING INTERFACE MOTION IN COMPRESSIBLE GAS DYNAMICS. W. Mulder and S. Osher, *Department of Mathematics, University of California, Los Angeles, California 90024, USA*; James A. Sethian, *Department of Mathematics, University of California, Berkeley, California 94720, USA.*

A "Hamilton–Jacobi" level set formulation of the equations of motion for propagating interfaces has been introduced recently by Osher and Sethian. This formulation allows fronts to self-intersect, develop singularities, and change topology. The numerical algorithms based on this approach handle topological merging and breaking naturally, work in any number of space dimensions, and do not require that the moving front be written as a function. Instead, the moving front is embedded as a particular level set in a fixed domain partial differential equation. Numerical techniques borrowed from hyperbolic conservation laws are then used to accurately capture the complicated surface motion that satisfies the global entropy condition for propagating fronts given by Sethian. In this paper, we analyze the coupling of this level set formulation to a system of conservation laws for compressible gas dynamics. We study both conservative and non-conservative differencing of the level set function and compare the two approaches. To illustrate the capability of the method, we study the compressible Rayleigh–Taylor and Kelvin–Helmholtz instabilities for air–air and air–helium boundaries. We perform numerical convergence studies of the method over a range of parameters and analyze the accuracy of this approach applied to these problems.

A HIGH-RESOLUTION EULER SOLVER BASED ON MULTIGRID, SEMI-COARSENING, AND DEFECT CORRECTION. Wim A. Mulder, *Department of Mathematics, University of California, Los Angeles, California 90024-1555, USA.*

In an earlier paper, an $O(N)$ method for the computation of stationary solutions to the Euler equations of inviscid compressible gas dynamics has been described. The method is a variant of the multigrid technique and employs semi-coarsening in all co-ordinate directions simultaneously. It provides good convergence rates for first-order upwind discretisations even in the case of alignment, the flow being aligned with the grid. Here we discuss the application of this scheme to higher-order discretisations. Two-grid analysis for the linear constant-coefficient case shows that it is difficult to obtain uniformly good convergence rates for a higher-order scheme, because of waves perpendicular to stream lines. The defect correction technique suffers from the same problem. However, convergence to a point where the residual of the total error (the sum of the iteration error and the discretisation error) is of the order of the truncation error can be obtained in about seven defect correction cycles, according to estimates for the linear constant-coefficient equations. This result is explored for the nonlinear case by some illustrative numerical experiments.

EVOLUTION OF SCALAR FIELDS FROM CHARACTERISTIC DATA. R. Gomez and J. Winicour, *Department of Physics and Astronomy, University of Pittsburgh, Pittsburgh, Pennsylvania 15260, USA*; R. Isaacson, *Physics Division, National Science Foundation, Washington, DC 20550, USA.*

We present a new algorithm for solving nonlinear wave equations when initial data is specified on characteristic surfaces. The algorithm is directly applicable to hyperbolic systems such as Maxwell, Yang–Mills, and gravitational fields. The basic principles should also be applicable to hydrodynamics. It is an especially effective approach for studying radiation fields. We show that this method is stable, globally convergent to second order in the grid spacing, and satisfies an energy conservation law. We



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carry out numerical studies of scalar wave equations with nonlinear self-interactions for some examples of physical interest. We observe nonlinear phenomena such as backscattering, radiative tail decay, and approximate analogues to solitons in three-dimensions.

A DOMAIN DECOMPOSITION METHOD FOR GENERATING ORTHOGONAL POLYNOMIALS FOR A GAUSSIAN WEIGHT ON A FINITE INTERVAL. Raymond C. Y. Chin, *Lawrence Livermore National Laboratory, Livermore, California 94550, USA.*

A domain decomposition method has been developed for generating orthogonal polynomials for a Gaussian weight on $(-1, 1)$. The method takes advantage of the underlying asymptotic structure of the orthogonal polynomials and, hence, it is *effective* in the sense that it makes maximal use of the analytic properties of the solution to increase accuracy and efficiency. These polynomials are necessary for constructing Gaussian quadrature formulas that are encountered in large quantum chemistry computational packages and in calculating the Compton scattering kernel and its associated angular moments.

AUTOMATED ANGULAR MOMENTUM RECOUPLING ALGEBRA. H. T. Williams, *Department of Physics, Washington and Lee University, Lexington, Virginia 24450, USA*; Richard R. Silbar, *Theoretical Division, Los Alamos National Laboratory, Los Alamos, New Mexico 87545, USA.*

We present a set of heuristic rules for algebraic solution of angular momentum recoupling problems. The general problem reduces to that of finding an optimal path from one binary tree (representing the angular momentum coupling scheme for the reduced matrix element) to another (representing the sub-integrals and spin sums to be done). The method lends itself to implementation on a microcomputer, and we have developed such an implementation using a dialect of LISP. We describe both how our code, called RACAH, works and how it appears to the user. We illustrate the use of RACAH for several transition and scattering amplitudes matrix elements occurring in atomic, nuclear, and particle physics.

A FRONT-TRACKING METHOD FOR VISCOUS, INCOMPRESSIBLE, MULTI-FLUID FLOWS. Salih Ozen Unverdi and Gretar Tryggvason, *Department of Mechanical Engineering and Applied Mechanics, The University of Michigan, Ann Arbor, Michigan 48109, USA.*

A method to simulate unsteady multi-fluid flows in which a sharp interface or a front separates incompressible fluids of different density and viscosity is described. The flow field is discretized by a conservative finite difference approximation on a stationary grid, and the interface is explicitly represented by a separate, unstructured grid that moves through the stationary grid. Since the interface deforms continuously, it is necessary to restructure its grid as the calculations proceed. In addition to keeping the density and viscosity stratification sharp, the tracked interface provides a natural way to include surface tension effects. Both two- and three-dimensional, full numerical simulations of bubble motion are presented.

UPWIND RELAXATION METHODS FOR THE NAVIER–STOKES EQUATIONS USING INNER ITERATIONS. Arthur C. Taylor, III, *Department of Mechanical Engineering and Mechanics, Old Dominion University, Norfolk, Virginia 23529-0247, USA*; Wing-fai Ng, *Mechanical Engineering Department, Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061-0238, USA*; Robert W. Walters, *Department of Aerospace and Ocean Engineering, Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061-0238, USA.*

An upwind line relaxation algorithm for the Navier–Stokes equations which employs inner iterations is applied to a supersonic and a subsonic test problem. The purpose of using inner iterations is to accelerate the convergence to steady-state solutions, thereby reducing the overall CPU time. A con-

vergence criterion is developed to assist in automating the inner iterative procedure. The ability of the line inner iterative procedure to mimic the quadratic convergence of the direct solver method is confirmed in both test problems, but some of the non-quadratic inner iterative results were more efficient than the quadratic results. In the supersonic test case, the use of inner iterations was very efficient in reducing the residual to machine zero. For this test problem, the inner iteration method required only about 65% of the CPU time which was required by the most efficient line relaxation method without inner iterations. In the subsonic test case, poor matrix conditioning forced the use of under-relaxation in order to obtain convergence of the inner iterations, resulting in an overall method which was less efficient than line relaxation methods which employ a more conventional CPU savings strategy.